Marking Schemes

The examination emphasises the testing of understanding, the application of knowledge and the use of processing skills. Candidates are advised to study this document in conjunction with the examiner's comments on candidates' performance in this booklet.

In answering questions, candidates are expected to demonstrate an understanding of the question, an ability to deploy relevant knowledge in response to the question, and to present their answers logically and coherently.

Advanced Level
Paper I Section A

1. (a) (i) Input power = \(15 \text{ kg s}^{-1} \times 10 \text{ m s}^{-2} \times 12 \text{ m} \times 90\%\)
   \[= 1620 \text{ W}\]

   (ii) \[P = \tau \cdot \omega\]
   \[= 160 \text{ N m} \times 7.85 \text{ rad s}^{-1}\]
   \[= 1256 \text{ W}\]
   Efficiency = \[
   \frac{1256 \text{ W}}{1620 \text{ W}} \times 100\% \]
   \[= 77.5\%\]

   (b) (i) \[\omega_1 r_1 = \omega_2 r_2\]
   \[\omega_2 = 7.85 \text{ rad s}^{-1} \times \frac{0.80 \text{ m}}{0.02 \text{ m}}\]
   \[= 314 \text{ rad s}^{-1} \text{(i.e. 50 rev s}^{-1}\text{)}\]

   (ii) Let \(T\) be the tension at \(B\).
   \[(T - 50 \text{ N}) \times 0.80 \text{ m} = 160 \text{ N m}\]
   \[T = 250 \text{ N}\]

2. (a) (i) Energy is needed or work has to be done to move an object to infinity (the reference point where the p.e. is taken to be zero).

   (ii) Field strength \[g = -\frac{\Delta V}{\Delta r} = \frac{[-13.34 - (-20.01)] \times 10^6 \text{ J kg}^{-1}}{(30 - 20) \times 10^6 \text{ m}}\]
   \[= -0.667 \text{ N kg}^{-1} \text{(towards the earth's centre)}\]

   (b) (i) \[
   \frac{GMm}{r^2} = m \frac{v^2}{r}\]
   \[
   \frac{GM}{r} = v^2\]
   \[v^2 = 13.34 \times 10^6\]
   \[v = 3.65 \times 10^3 \text{ m s}^{-1}\]

   (ii) At \(A\),
   \[\text{P.E. + K.E.} = (-62.53 \times 10^6 \text{ J kg}^{-1})(5.0 \times 10^4 \text{ kg}) + 0\]
   \[= -3.1265 \times 10^{12} \text{ J}\]

   At \(B\),
   \[\text{P.E. + K.E.} = (-13.34 \times 10^6 \text{ J kg}^{-1})(5.0 \times 10^4 \text{ kg}) + \frac{1}{2}(5.0 \times 10^4 \text{ kg})(3.652 \times 10^3 \text{ m s}^{-1})^2\]
   \[= -3.335 \times 10^{11} \text{ J}\]
   \[\Delta (\text{P.E. + K.E.}) = 2.79 \times 10^{12} \text{ J} \quad (\Delta \text{P.E.} = 2.46 \times 10^{12} \text{ J}, \Delta \text{K.E.} = 3.34 \times 10^{11} \text{ J})\]

3. (a) (b)  

4. (i)
(b) \(0 - \left(-\frac{GMm}{r}\right) = \frac{1}{2}mv'^2\)

\[
GM = \frac{1}{2} \nu'^2 \\
\nu'^2 = 2 \times 13.34 \times 10^6 \\
\nu' = 5.17 \times 10^3 \text{ m s}^{-1}
\]

(iv) P.E. + K.E. \(= -\frac{GMm}{r} + \frac{1}{2}mv^2\)

\[
\frac{1}{2}mv^2 \left(\because \frac{GMm}{r^2} = \frac{mv^2}{r}\right)
\]

Due to the resistance, the total mechanical energy \(-\frac{1}{2}mv^2\) decreases and therefore \(v\) increases.

3. (a) The slide-wire is uniform in both cross-section and resistivity.

(b) No, the length \(L\) is measuring the p.d. across the resistance box or the terminal p.d. of the battery.

Or No, it is measuring \(E - Ir\) since there is internal resistance \(r\).

(c) \(IR = kL\) where \(k\) is a constant because \(V_{PQ} = IR = V_{AC} \propto L\)

\[
E = I \left(r + R\right) \\
E = \frac{rkL}{R} + kL
\]

\[
\frac{E}{k} = \frac{1}{R} + \frac{1}{R} \\
\frac{1}{R} = \text{constant} \times \frac{1}{L} - \frac{1}{r}
\]

(d) The negative intercept on the \(\frac{1}{R}\) axis = 0.010 \(\Omega^{-1}\) (accept 0.0096 \(\Omega^{-1}\) – 0.0104 \(\Omega^{-1}\))

The internal resistance of the battery \(r = \frac{1}{0.01} = 100 \Omega\)

(e) Unchanged.

The p.d. \(V_{AC}\) corresponds to the same \(L\).

4. (a) \(E = \frac{V}{d}\)

\[
= \frac{320 \text{ V}}{0.016 \text{ m}} \\
= 2.0 \times 10^4 \text{ V m}^{-1} \text{ or } \text{N C}^{-1}
\]

(b) (i) \(320 \text{ V} \quad 1.6 \text{ cm} \quad 10 \text{ cm}\)

\(v = 3.7 \times 10^7 \text{ m s}^{-1}\)
4. (b) (i) 
\[ a = \frac{eE}{m} = \frac{1.60 \times 10^{-19} \text{C} \times 2.0 \times 10^4 \text{NC}^{-1}}{9.11 \times 10^{-31} \text{kg}} \]
\[ = 3.513 \times 10^{15} \text{m s}^{-2} \]
\[ y = \frac{1}{2} at^2 \text{ and } x = vt \]

When \( y = 0.008 \text{ m} = \frac{1}{2} at^2 = \frac{1}{2} (3.513 \times 10^{15} \text{ m s}^{-2}) t^2 \)
\[ t = 2.134 \times 10^{-9} \text{ s} \]
\[ x = vt = (3.7 \times 10^7 \text{ m s}^{-1})(2.134 \times 10^{-9} \text{ s}) \]
\[ d = 0.079 \text{ m or 7.9 cm} \]

(ii) 
\[ d \propto \frac{1}{\sqrt{V}} \]
\[ d = \frac{7.9 \text{ cm}}{\sqrt{2}} = 5.6 \text{ cm or 0.056 m} \]

(c) 
\[ C = \frac{\epsilon_o A}{d} = \frac{Q}{V} \quad \text{i.e. } Q = \frac{V}{d} \epsilon_o A \]
\[ Q = 2.0 \times 10^4 \text{ V m}^{-1} \times 8.85 \times 10^{-12} \text{ F m}^{-1} \times (0.1 \text{ m})^2 \]
\[ = 1.77 \times 10^{-9} \text{ C} \]

(d) 
\[ eE = evB \]
\[ 2.0 \times 10^4 \text{ V m}^{-1} = 3.7 \times 10^7 \text{ m s}^{-1} \times B \]
\[ B = 5.41 \times 10^{-4} \text{ T or Wb m}^{-2} \]

5. (a) (i) 
\[ \frac{pV}{n} = nRT \]
\[ (1.00 \times 10^5 \text{ Pa})(300 \times 10^{-6} \text{ m}^3) = n (8.31 \text{ J mol}^{-1} \text{ K}^{-1}) (273 + 27 \text{ K}) \]
\[ n = 0.012 \text{ (mole)} \]

(ii) 
\[ PV = \frac{1}{3} Nmc^2 \]
\[ \sqrt{c^2} = \sqrt{\frac{3 (1.00 \times 10^5 \text{ Pa})(300 \times 10^{-6} \text{ m}^3)}{(0.012 \text{ mol})(4.00 \times 10^{-3} \text{ kg mol}^{-1})}} \]
\[ = 1.369 \times 10^3 \text{ m s}^{-1} \]

(b) (i) 
\[ \frac{V_1}{T_1} = \frac{V_2}{T_2} \]
\[ \frac{20 \text{ cm}}{(273 + 27 \text{ K})} = \frac{h}{(273 + 90 \text{ K})} \]
\[ h = 24.2 \text{ cm} \]

(ii) 
\[ \frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \text{(or } p_1 V_1 = p_2 V_2) \]
\[ 1.00 \times 10^5 \text{ Pa} = \frac{p_2}{(273 + 27 \text{ K})} \]
\[ p_2 = 1.21 \times 10^5 \text{ Pa} \]
From $B$ to $C$ (process in (b)(ii)), the temperature is constant and thus the internal energy remains unchanged, work is done on the gas and according to $\Delta U = Q + W$ there is heat transferred from the gas.

(iv) $\Delta p = \frac{mg}{A}$

$\frac{(1.21-1.00)\times10^5 \text{ Pa} \times \frac{300}{20} \times 10^{-4} \text{ m}^2}{10 \text{ m s}^{-2}}$

$m = 3.15 \text{ kg}$

Paper I Section B

6. (a) $mg = kx_o$

$x_o = \frac{(0.01 \text{ kg})(10 \text{ m s}^{-2})}{5 \text{ N m}^{-1}}$

$= 0.02 \text{ m}$

$mg x_i = \frac{1}{2} kx_i^2$

$x_i = \frac{2 (0.01 \text{ kg})(10 \text{ m s}^{-2})}{5 \text{ N m}^{-1}}$

$= 0.04 \text{ m}$

(b)
6. (c) \[ mg - kx = m\ddot{x} \]
\[ kx_0 - kx = m\ddot{x} \quad (\therefore mg = kx_0) \]
\[ -k(x - x_0) = m\ddot{x} \]

Simple harmonic motion.

[Or Put \( y = x - x_0 \), \( mg - k(y + x_0) = m\ddot{y} \]
\[ -ky = m\ddot{y} \quad (\therefore mg = kx_0) \]

(d) At the lowest position of the s.h.m., maximum supporting force equals

\[ kx_1 = (5 \text{ N m}^{-1})(0.04 \text{ m}) \]
\[ = 0.2 \text{ N} \]

(e) Amplitude increases (doubled) because equilibrium position lowered.

Frequency of oscillation decreases (by a factor of \( \frac{1}{\sqrt{2}} \)) as the mass of the system increases (doubled). \( \omega^2 = \frac{k}{m} \)

7. (a) [Diagram of collimator telescope and cross hairs]

(b) (i) Reading: 273°39' or 273.65°
\[ \theta = (273.65° - 224.50°) / 2 = 24.58° \]

Percentage error in \( \theta \)
\[ = \left[ \frac{1^\circ}{60} \times 24.58^\circ \right] \times 100\% = 0.07\% \]

(ii) Eliminate the systematic error, if any, associated with the central position reading.
Or Reduce the percentage error in \( \theta \)

(c) (i) 

<table>
<thead>
<tr>
<th>First-order diffraction angle ( \theta )</th>
<th>( \lambda ) (nm)</th>
<th>( \sin \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>24.58°</td>
<td>656.3</td>
</tr>
<tr>
<td>Cyan</td>
<td>17.94°</td>
<td>486.1</td>
</tr>
<tr>
<td>Blue</td>
<td>15.96°</td>
<td>434.1</td>
</tr>
<tr>
<td>Violet</td>
<td>15.07°</td>
<td>410.2</td>
</tr>
</tbody>
</table>

Axes labelled with appropriate scales
Points correctly plotted
Correct graph
A plot of $\lambda$ against $\sin \theta$.

Grating spacing = slope of the graph ($= \frac{1}{\text{slope}}$ if $\sin \theta$ Vs $\lambda$ is plotted)

$$= 1571 \text{ nm} \pm 30 \text{ nm}$$

(ii) $$\sin \theta = \frac{n \lambda}{d} = n \frac{410.2}{1571} \leq 1$$

$n \leq 3.83$, highest order is 3.

(d) $E = \hbar \frac{c}{\lambda} = 6.63 \times 10^{-34} \text{ J s} \frac{3 \times 10^8 \text{ m s}^{-1}}{410.2 \times 10^{-9} \text{ m}} = 4.84 \times 10^{-19} \text{ J or 3.03 eV}$

$\therefore$ Violet from $-3.40 \text{ eV} + 3.03 \text{ eV}$ (i.e. $n = 6$)

$$= -0.37 \text{ eV}$$

(i.e. $n = 6$)
8. (a) (i) \( V = IR \)
\[ 3 \text{V} = I_1 \times 1 \Omega \]
\[ I_1 = 3 \text{A} \]

Mention \( V_C = 0 \) or capacitor behaves like shorted at \( t = 0 \). (The induced e.m.f. in the inductor will oppose the current change and initially current only flows in the branch with the uncharged capacitor.)

(ii) \( V = IR \)
\[ 3 \text{V} = I_2 \times 1.5 \Omega \]
\[ I_2 = 2 \text{A} \]

At steady state, inductor has only resistance, no inductance or inductive reactance or back e.m.f. (After a few minutes, the capacitor will be fully charged and a steady current flows only in the branch with the inductor which does not oppose the current at the steady state.)

(b) (i) At the steady state,
\[ 3 \text{V} = I_2 \times 1 \Omega + V_C \]
\[ 3 \text{V} = 2 \text{A} \times 1 \Omega + V_C \]
\[ V_C = 1 \text{V} \]

(ii) At the steady state,
\[ \frac{1}{2} L I^2 = \frac{1}{2} (54 \times 10^{-3} \text{H})(2 \text{A})^2 \]
\[ = 0.108 \text{J} \]

(iii) \( f = 100 \text{Hz}, T = 0.01 \text{s} \)

(c) Replace the capacitor with a neon lamp.
Opening the switch causes the current to fall very rapidly to zero and the rate of change of flux is large. The induced e.m.f. is therefore large and is sufficiently great (more than 50 V) to produce a brief flash of the neon lamp.

(Or Remove the capacitor and sparks will be observed when the switch is opened.)
Marks

9. (a) (i) \[ V_o = A_o (V_2 - V_1) \]
\[ 15 \text{ V} = A_o (160 \mu \text{V}) \]
\[ A_o = 9.375 \times 10^4 \]

1 2

(ii) (I) \[ \mathcal{X} : \text{thermistor} \]
\[ R \text{ and the thermistor forms a potential divider. When temperature rises, the resistance} \]
\[ \text{of the thermistor decreases until } V_- \text{ falls below } V^+, \text{ the op amp (comparator) gives a} \]
\[ \text{high output (saturates) and turns on the LED.} \]

1

1+1 3

(II) Shift the sliding contact of the potential divider \[ Y \] towards the high potential end (+15V).

1 1

1 2

(b) (i) Non-inverting amplifier.

\[ R_1 \text{ and } R_2 \text{ combination provides negative feedback to the op amp / Feed back part of the} \]
\[ \text{output to the inverting input } V_- \text{ of the op amp.} \]

1 2

(ii)
\[ V_- = V_o \frac{R_2}{R_1 + R_2} \]
\[ \text{And } V_- \approx V_o = V_m \]

1 1

\[ \therefore \]
\[ V_m = V_o \frac{R_2}{R_1 + R_2} \]
\[ \frac{V_o}{V_m} = 1 + \frac{R_1}{R_2} \]

1 2

(iii) \[ R_1 = 100 \ \text{k}\Omega; \ R_2 = 4.7 \ \text{k}\Omega \]
\[ \text{gain : } 1 + \frac{R_1}{R_2} = 22.3 \]

1+1 3

(c) - the gain is predictable and more or less independent of the characteristics of the op amp
- greater stability
- less distortion of output, i.e., the amplification is more linear (accept not saturate easily)
- the gain is constant over a wide range of frequency

\[ \text{ANY} \]
\[ \text{TWO} \]
\[ \text{AT1} \]

2 2

1

1

1 3